LECTURE 4: GRAVITY ANOMALIES AND ISOSTASY

Average gravity on the Earth's surface is about g=9.8 m/s^2, and varies by ~5300 mgal (about 0.5% of g) from pole to equator. (1 mgal=10^{-5} m/s^2) Gravity anomalies are local variations in gravity that result from topographic and subsurface density variations, and have amplitudes of several mgal and smaller.

Measurement of Absolute Gravity:

Pendulum Method: Measure the period $T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{L}{g}}$

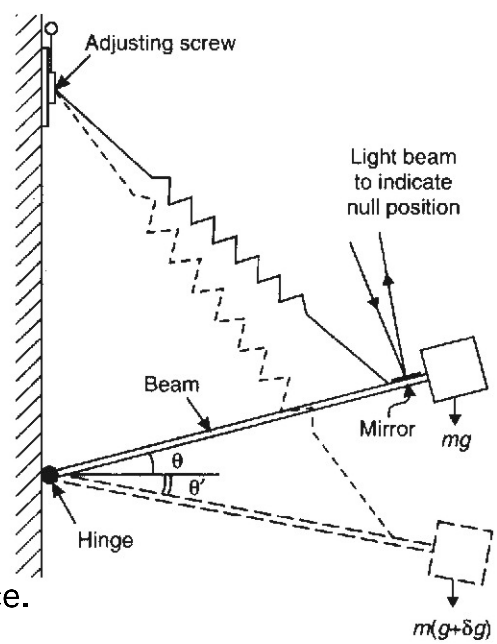
To measure 1 mgal variation, the period must be measured to within 1μs.

Free-fall Method: Measure the fall of a mass: $z = z_0 + ut + gt^2/2$

To measure 1 μgal variation, time must be measured to within 1ns.

Rise-and-fall Method: Measure time T for a thrown ball to rise and fall a height z: $z = g(T/2)^2/2$. Then $g = \frac{8(z_1 - z_2)}{(T_1^2 - T_2^2)}$. μgal precision; not portable.

Measurement of Relative Gravity:



Stable Gravimeter: Measure Δs, the change in a spring's length: $\Delta g = \frac{k}{m}\Delta s$

Unstable Gravimeter: Use a spring with built-in tension, so: $\Delta g = \frac{k}{m}s$

(LaCost-Romberg gravimeter)

Usage: Adjust the spring length to zero using a calibrated screw.

Sensitivity: 0.01 mgal for a portable device.

Superconducting Gravimeter: Suspend a niobium sphere in a stable magnetic field of variable strength. Sensitivity: 1 ngal

Gravity Corrections

Drift Correction: In relative gravity surveys, instrument drift can be corrected by making periodic measurements at a base station with known gravity.

Tidal Correction: Gravity changes during the day due to the tides in a known way. Tidal corrections can be computed precisely if time is known. For example, if the moon is directly overhead, the tidal correction would be:

$$\Delta g_T = G\frac{M_L}{r_L^2}\left(\frac{2R_E}{r_L} + 3\left(\frac{R_E}{r_L}\right)^2 + \ldots\right)$$ *This should be added to measured gravity*.

Eötvös Correction: Moving eastward at v_E, your angular velocity increases by: $\Delta\omega = v_E / (R_E \cos\lambda)$. This change increases the centrigugal acceleration: $\Delta a_C = \left(\frac{\mathrm{d}a_C}{\mathrm{d}\omega}\right)\Delta\omega = (2\omega R_E \cos\lambda)\left(\frac{v_E}{R_E \cos\lambda}\right) = 2\omega v_E$. Downward gravity changes by:

$\Delta g = -2\omega v_E \cos\lambda$. *The Eötvös effect decreases gravity when moving east*.

Latitude Correction: Absolute gravity is corrected by subtracting normal gravity on the reference ellipsoid: $g_n = g_e\left(1 + \beta_1 \sin^2\lambda + \beta_2 \sin^4 2\lambda\right)$

where $g_e = 9.780327$ m/s^2, $\beta_1 = 5.30244 \times 10^{-3}$, and $\beta_2 = -5.8 \times 10^{-6}$.

Relative gravity is corrected by differentiating g_n with respect to λ:

$\Delta g_{\mathrm{lat}} = 0.8140 \sin 2\lambda$ mgal per km north-south displacement. *This correction is subtracted from stations closer to the pole than the base station*.

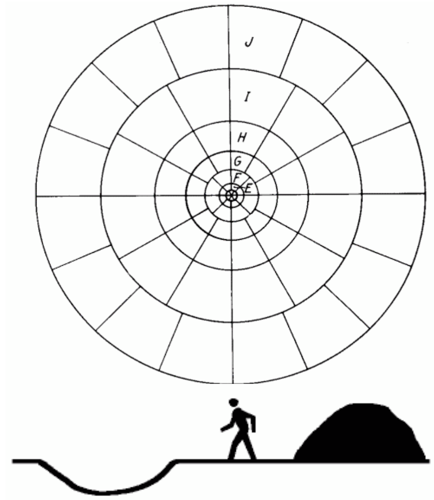
Terrain Correction: Nearby topography perturbs gravity measurements upward due to mass mass excess above the station (nearby hills) or due to mass deficiency below the station (nearby valleys). The terrain correction is computed using:

$\Delta g = G\left(\mathrm{d}m\cos\theta\right)/\left(r^2 + z^2\right)$

where r and z are the horizontal and vertical distances to dm, and θ is the angle to the vertical. *The terrain correction is always positive*.

Integrating over a sector gives:

$\Delta g_T = G\rho\phi\left(\left(\sqrt{r^2 + h^2} - r_1\right) - \left(\sqrt{r^2 + h^2} - r_2\right)\right)$

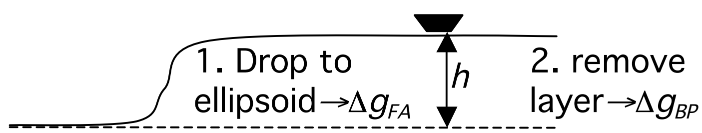


r_1 and r_2 are the inner and out radii, h is the height, ϕ is the sector angle.

Bouguer Plate Correction: This correction compensates for a rock layer of thickness h between the measurement elevation level and the reference level. For a solid disk of density ρ and radius r, the terrain correction is: $\Delta g_T = 2\pi G\rho\left(h - \left(\sqrt{r^2 - h^2} - r\right)\right)$. Allowing r to become infinite, we obtain:

$\Delta g_{BP} = 2\pi G\rho h = 0.0419 \times 10^{-3} \rho$ mgal/m if ρ is in kg/m^3.

This correction must be subtracted, unless the station is below sea level in which case a layer of rock must be added to reach the reference level.



For gravity measured over water, water must be replaced with rock by assigning a slab with density $\left(\rho_{\text{rock}} - \rho_{\text{water}}\right)$.

Free-air Correction: This correction compensates for gravity's decrease with distance from the Earth's surface. It is determined by differentiating g:

$$\Delta g_{FA} = \frac{\partial}{\partial r}\left(-G\frac{M_E}{r^2}\right) = +2G\frac{M_E}{r^3} = -\frac{2}{r}g = 0.3086 \text{ mgal/m}$$

This correction must be added (for stations above sea level).

Combined Correction: Free air and Bouguer corrections are often combined:

$$\Delta g_{FA} + \Delta g_{BP} = \left(0.3086 - 0.0419\rho \times 10^{-3}\right) \text{ mgal/m} = 0.197 \text{ mgal/m}$$

assuming a crustal density of 2670 kg/m^3. To obtain 0.01 mgal accuracy:

-- location must be known to within 10 m (for latitude correction)

-- elevation must be known to within 5 cm (for combined correction)

Geoid Correction: For long wavelength surveys, station heights must be corrected for the difference in gravity between the geoid height and the reference ellipsoid, which can vary spatially.

Density determination

Knowledge of the density of subsurface rocks is essential for the Bouguer and terrain corrections. Density can be measured in several ways:

- By measuring the density of rocks on the surface
- Using seismic velocity measurements (velocity increases with density)
- By applying the combined correction to depth variations in gravity measurements in a borehole. Assuming two measurements are separated by a height Δh and using the lower station as a reference level,

  The gravity correction at the upper borehole (free-air decreases gravity and Bouguer slab between the stations increases gravity) is:

$$\Delta g_{upper} = \Delta g_{FA} + \Delta g_{BP} = \left(0.3086 - 0.0419\rho \times 10^{-3}\right) \Delta h$$

  The gravity correction at the lower borehole (Bouguer slab between the stations decreases gravity) is:

$$\Delta g_{lower} = \Delta g_{FA} + \Delta g_{BP} = \left(0 + 0.0419\rho \times 10^{-3}\right) \Delta h$$

  Subtracting the two and solving for density ρ gives:

$$\rho = \left(3.683 - 11.93\,\Delta g / \Delta h\right) \times 10^{-3} \text{ km/m}^3$$

Gravity Anomalies

After the appropriate corrections are applied, gravity data reveal information about subsurface density heterogeneity. How should this data be interpreted?

Gravity over a Uniform Sphere

Gravity for a sphere is the same as for a point mass. The z-component:

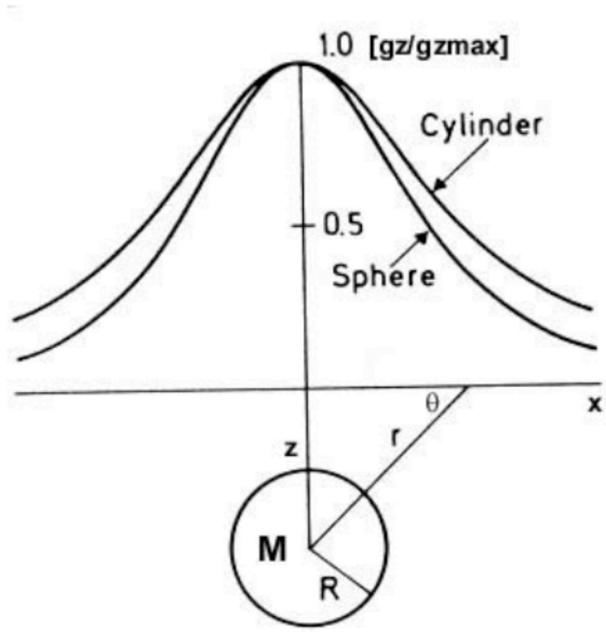
$\Delta g_z = \Delta g \sin\theta = G\frac{M}{r^2}\frac{z}{r}$ where

$M = \frac{4\pi}{3}R^3\Delta\rho$ and $r^2 = z^2 + x^2$ giving:

$$\Delta g_z = \frac{4\pi}{3}G\left(\frac{\Delta\rho R^3}{z^2}\right)\left(\frac{z^2}{z^2+x^2}\right)^{3/2}$$

The maximum is at x=0, where:

$$\Delta g_{z\max} = \frac{4\pi}{3}G\left(\frac{\Delta\rho R^3}{z^2}\right)$$



Rule of thumb: $z = 0.65w$ where w is the width at half height of the anomaly.

Gravity over an Infinite Line

An infinitely long line of mass m per unit length produces a gravity anomaly:

$\Delta g_z = \frac{2Gmz}{z^2 + x^2}$ where z and x are the vertical and horizontal distances to the line.

Gravity over an Infinite Cylinder

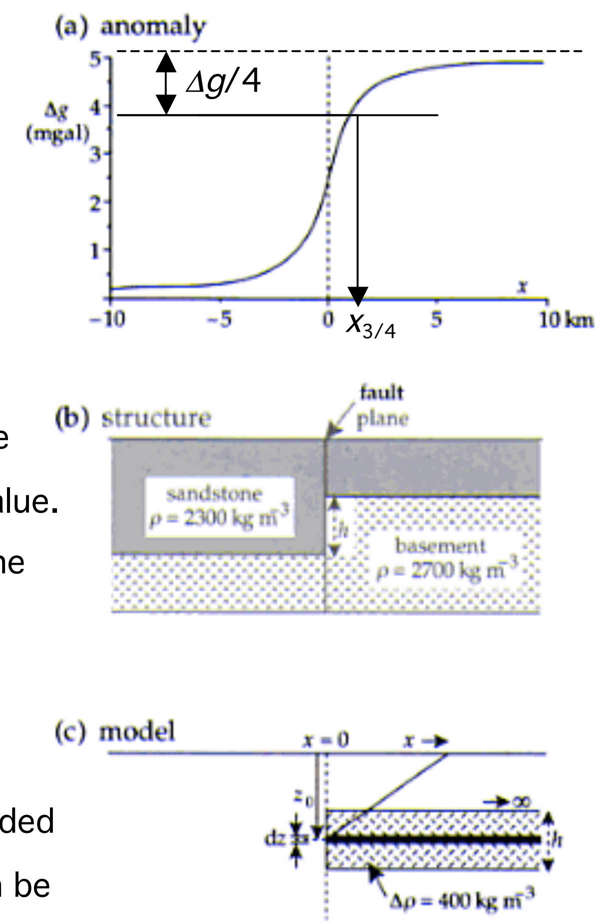
An infinitely long cylinder is a useful analogue for a buried syncline or anticline.

$\Delta g_z = 2\pi G\left(\frac{\Delta\rho R^2}{z}\right)\left(\frac{z^2}{z^2+x^2}\right)$ and $\Delta g_{z\max} = 2\pi G\left(\frac{\Delta\rho R^2}{z}\right)$

Rule of thumb: $z = 0.5w$. A structure must be more than 20× longer than it is wide or deep for the "infinite" approximation to be valid (ignore edge effects).

Gravity over a Semi-Infinite Horizontal Sheet

A horizontally truncated thin sheet can be used to approximate a bedded formation offset by a fault. If the fault is centered at x=0, z_0=0, then the gravity anomaly is: $\Delta g_z = 2G\Delta\rho h\left(\frac{\pi}{2}+\tan^{-1}\left(\frac{x}{z_0}\right)\right)$

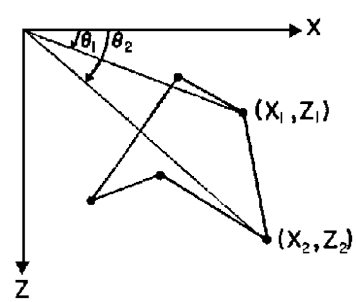


Rule of thumb: $z_0 \sim x_{1/4} \sim x_{3/4}$

Where $x_{1/4}$ and $x_{3/4}$ are the positions where the gravity anomaly is ¼ and ¾ its max value. Note that as x→∞, $\Delta g_z = 2G\Delta\rho h$, which is the solution for a Bouguer Plate anomaly.

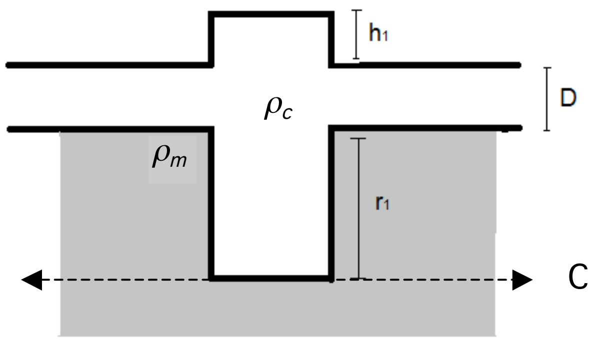
Gravity anomaly of arbitrary shape

Any shape can be approximated as an n-sided polygon, the gravity anomaly of which can be computed using Talwani's algorithm. This algorithm estimates gravity by computing a line integral around the perimeter: $\Delta g_z = 2G\Delta\rho\oint z\,d\theta$

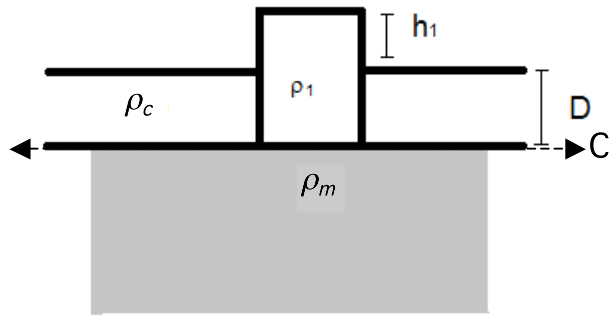


Isostasy

Long wavelength variations in topography are isostatically compensated at depth. This means that the excess mass in positive topography is compensated by a mass deficiency at depth. There are three types of isostasy.



Airy Isostasy



Pratt Isostasy

Airy Isostasy:

Lateral variations in crustal thickness allow surface topography to be compensated by a deep crustal root. The thickness of this root is determined by requiring the mass in columns above the compensation depth (C) to be equal:

$$r_1 = \frac{\rho_c}{\rho_m - \rho_c} h_1 \quad \text{or, if the topography is under water,} \quad r_1 = \frac{\rho_c - \rho_w}{\rho_m - \rho_c} h_1$$

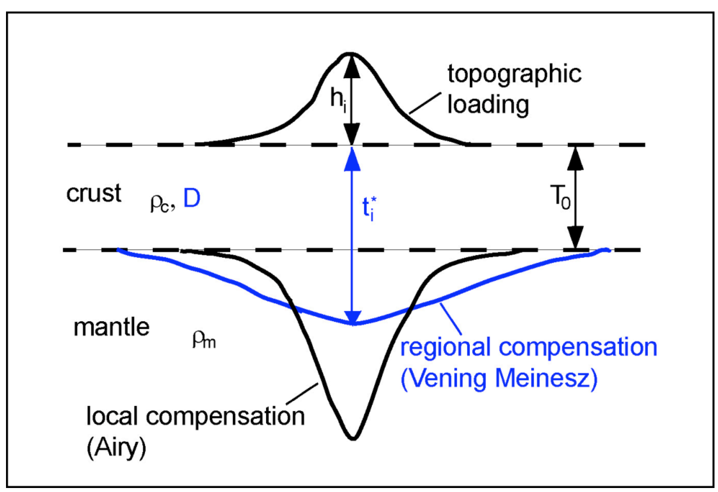
Pratt Isostasy:

Lateral variations in crustal density compensate topography, so again the mass in columns above the compensation depth (C) are equal. The density is:

$$\rho_1 = \frac{D}{h_1 + D} \rho_c \quad \text{or, if the column is a depth } d \text{ under water,} \quad \rho_0 = \frac{\rho_c D - \rho_w d}{D - d}$$

Vening Meinesz Isostasy:

In this type of isostasy, short-wavelength topography is supported by the elastic strength of the crustal rocks. The load is instead distributed by the bent plate over a broad area. This distributed load is compensated.



Gravity Anomalies over Topography

Uncompensated topography (Short-wavelengths)

Free-air anomaly (apply the free-air correction only):

$\Delta g+\Delta g_{FA}$ >> 0 because of the topography's excess mass

Bouguer anomaly (apply both free-air and Bouguer plate corrections):

$\Delta g+\Delta g_{FA}-\Delta g_{BP} \sim 0$ because Bouguer corrects for excess mass.

Compensated topography (Long-wavelengths)

Free-air anomaly (apply the free-air correction only):

$\Delta g+\Delta g_{FA} \sim 0$ because topography is compensated (no excess mass)

Bouguer anomaly (apply both free-air and Bouguer plate corrections):

$\Delta g+\Delta g_{FA}-\Delta g_{BP}$ << 0 because Bouguer removes additional mass.

Undercompenstated topography: A too-shallow root, yields $\Delta g+\Delta g_{FA}>0$

Overcompensated topography: A too-shallow root, yields $\Delta g+\Delta g_{FA}<0$

Geoid Anomalies over Topography

The gravitational acceleration can be approximated as:

$g = -\frac{\partial U}{\partial r}\Delta U = \frac{\Delta U}{\Delta N}$ where ΔN is the change in geoid height

We can approximate the change in potential as:

$g\Delta N \sim \Delta U \sim -\int_0^{z_c} \Delta g dz = -\int_0^{z_c} 2\pi G z \Delta\rho(z) dz$ where z is positive downwards from the

surface to the compensation depth z_c

Then the geoid height can be written as:

$\Delta N \sim -\frac{2\pi G}{g_0}\int_0^{z_c} z\Delta\rho(z)dz$ This is non-zero because $\int_0^{h} \Delta\rho(z)dz = 0$ for isostasy.

Thus, the geoid anomaly should be positive over compensated positive topography (e.g., the continental lithosphere, mid-ocean ridges, Tibet, Andes)

The geoid gives better constraints on the depth-distribution of mass than does gravity.