

Spatial variations in the rate of sea level rise caused by the present-day melting of glaciers and ice sheets

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Abstract. The redistribution of surface water mass associated with the melting of glacial ice causes uplift near areas of mass depletion, depression of the seafloors, and changes in the earth's gravitational field which perturb the ocean surface. As a result, local spatial variations exist in the rate of sea level rise. Tide gauges on continental coastlines measure a sea level rise 5% smaller than the global average. Tide gauges in the hemisphere opposite a source of continental mass depletion measure sea level rise 10 to 20% greater than the global average produced by that source while satellites make measurements 10% too low. Because most long duration tide gauges are in the northern hemisphere, if the sources of sea level rise are unbalanced between the two hemispheres, estimates of global sea level rise could be in error by 10 to 20%. Individual tide gauges could be more seriously unrepresentative if they are near regions of significant present-day mass depletion.

Introduction

A century of tide gauge measurements indicates that global sea level is currently rising between 1 and 2 mm/yr [Douglas, 1991]. Of this, a significant portion must involve the melting of continental ice masses into the oceans [Warwick and Oerlemans, 1990]. The Earth's response to continental water-mass wastage is the immediate elastic vertical uplift of the crust around the depleted area. Vertical velocities near the Antarctic ice sheet would be a few mm/yr for a realistic scenario of mass depletion there [e.g., Hager, 1991; James and Ivins, 1995; Wahr et al., 1995]. The transfer of this mass to the oceans would similarly cause a smaller amplitude depression of the sea floors. In addition, the gravitational equipotential surface that defines sea level would be perturbed. As a result, the rate of sea level rise should vary spatially as both the sea surface and the sea floors move vertically at rates determined by the location of applied surface mass loads. Changes in sea level due to the elastic response of the earth to Quaternary climate change have been studied previously [e.g., Farrell and Clark, 1976]; our intent is to perform these calculations for plausible scenarios of present-day climate change.

Elastic Models of Sea Level Change

Changes in sea level are currently measured in two ways. Relative sea level is measured by tide gauges as the difference between the ocean-air and earth-ocean interfaces, both of which can move vertically as mass is redistributed. Geodetic satellites, such as TOPEX/POSEIDON or the Global Positioning System (GPS), make measurements relative to geodetic reference frames such as the International Terres-

trial Reference Frame, which can be tied to the center of mass or center of figure of the solid earth [e.g., Nerem, 1995]. The melting of continental ice into the oceans can cause both of these points to move in space relative to an inertial reference frame defined by the center of mass of the earth, ice and ocean system. In this reference frame, movements of the center of mass of the solid earth compensate for the movement of water mass on the surface. For simplicity, we calculate crustal motions and changes in "absolute" sea level, as they would be measured by satellites, in this inertial frame. This frame can be related to other geodetic frames through a rigid translation of the solid earth.

The expected rate of sea level rise can be calculated for a given loading scenario and elastic Earth model. The load, which varies with latitude, θ , and longitude, λ , causes a vertical displacement of the earth's surface, $U(\theta, \lambda)$, and a change in the earth's gravitational potential, $\Phi(\theta, \lambda)$. Both U and Φ can be calculated by convolving the load with the Green's functions for the earth's elastic response to a point load [Farrell, 1972]. The Green's functions are simply:

$$u(\psi) = \frac{a}{M_B} \sum_{n=0}^{\infty} h_n P_n(\cos \psi) \quad (1)$$

for vertical displacement and

$$\phi(\psi) = \frac{ag}{M_B} \sum_{n=0}^{\infty} (1 + k_n) P_n(\cos \psi) \quad (2)$$

for a change in gravitational potential [Farrell and Clark, 1976]. Here, ψ is angular distance, and a , M_B , and g are the earth's radius, mass, and surface gravitational acceleration. The elastic love numbers, h_n and k_n , describe, for each harmonic degree n , the vertical displacement and potential change which arise from the imposition of a point load at $\psi = 0$. Included in (2) is the potential change due to the load itself, $ag/M_B \sum_{n=0}^{\infty} P_n(\cos \psi)$.

Special attention must be paid to the degree one terms, because they typically include a shift in the center of mass of the earth [e.g., Farrell, 1972]. In the inertial frame, the center of mass of the earth plus load cannot move in space, so the degree one term of the potential in (2) is zero. Farrell [1972] gives elastic love numbers in a reference frame defined after the application of the load, so we include a rigid translation of the solid earth such that the net change in the center of mass of the earth-load system is zero. Farrell [1972] gives a degree one potential love number of $k_1 = 0$; we require $k_1 = -1$ for the degree one term of (2) to be zero. This change must be accompanied by a corresponding change of $h_1 = -0.290$ to $h_1 = -1.290$. In this inertial frame, the elastic earth moves rigidly in space away from a positive load applied at the earth's surface. We have recalculated $u(\psi)$ and $\phi(\psi)$ using love numbers and methods of summation given by Farrell [1972].

The melting of ice into the oceans causes local changes in absolute sea level, $S_A(\theta, \lambda)$, which depend upon variations

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in the local gravitational potential, $\Phi(\theta, \lambda)$. The average change in sea level measured in the inertial frame is given by the average height of sea water added, S_V , and the change in ocean basin volume produced by depression of the sea floors, $\langle U(\theta, \lambda) \rangle_O$, where $\langle \rangle_O$ indicates an average over the ocean area. Then,

$$S_A(\theta, \lambda) = S_V + \Phi(\theta, \lambda) - \langle \Phi(\theta, \lambda) \rangle_O + \langle U(\theta, \lambda) \rangle_O \quad (3)$$

We have subtracted the term $\langle \Phi(\theta, \lambda) \rangle_O$ in order to choose a potential surface which conserves volume.

Relative sea level rise, S_R , is absolute sea level rise measured relative to the surface of the solid earth, so we subtract the local vertical displacement, $U(\theta, \lambda)$, from S_A :

$$S_R(\theta, \lambda) = S_V + \Phi(\theta, \lambda) - \langle \Phi(\theta, \lambda) \rangle_O + \langle U(\theta, \lambda) \rangle_O - U(\theta, \lambda) \quad (4)$$

Note that $\langle S_R \rangle_O = S_V$; the change in ocean basin volume is not observed in global averages of relative sea level. Because S_R and S_A differ by $U(\theta, \lambda)$, these quantities exhibit different spatial patterns of sea level rise.

Because both $U(\theta, \lambda)$ and $\Phi(\theta, \lambda)$ determine both absolute and relative sea level, which in turn involves a redistribution of mass within the oceans, we solve (3) and (4) by an iterative process. The mass redistribution within the oceans is calculated for each step, as are the resulting values of $U(\theta, \lambda)$ and $\Phi(\theta, \lambda)$. The sequence converges rapidly; we performed three iterations and determined sea level to within a few tenths of a percent. We used a load resolution of $\Delta\theta = \Delta\lambda = 0.5$ degrees, and an ocean grid provided by the *U.S. Navy Global Elevation Data* [1984].

The Elastic Response to Sea Level Rise

As our first case, we add water to the ocean basins and let the ocean surface redistribute itself according to the resulting elastic surface deformation and gravitational potential. In doing so, we ignore the continental sources of this water. The values of $U(\theta, \lambda)$, $S_A(\theta, \lambda)$, and $S_R(\theta, \lambda)$ (Fig. 1), are directly proportional to the average height of water added to the oceans, so we express them as percentages of S_V .

The geometry of the ocean basins on the earth is such that the center of mass of water added to the oceans is located beneath the southern Pacific. To prevent a net shift in the center of mass of the earth-load system, the solid earth moves toward a point in Asia (28.9°N, 45.2°E) by 6.2% of S_V . This degree one component of vertical displacement causes the southern Pacific seafloor to sink and Asian crust to rise in the inertial reference frame (Fig. 1a). The higher order degrees contribute significantly as well, as shown by the general depression of the ocean basins. In this case, the average displacement of the solid earth over the ocean area is -8.3% of S_V . As a result, the average absolute sea level change, $\langle S_A \rangle_O$, is only 91.7% of what it would be on a rigid earth. In fact, $S_A < S_V$ everywhere on the earth (Fig. 1b).

Because absolute sea level is measured in an inertial reference frame, it has zero degree one component. Variations in the higher order degrees cause the sea surface to rise unevenly. The additional sea water increases the gravitational potential in the ocean interiors, thus raising their sea surface relative to neighboring coastal areas. This effect is diminished in the Pacific where the potential is decreased due to the movement of the solid earth.

Relative sea level (Fig. 1c), is the difference between absolute sea level (Fig. 1b) and vertical displacement (Fig. 1a). In general, coastlines show less than average sea level rise, and open oceans more. The Pacific deepens 5% faster

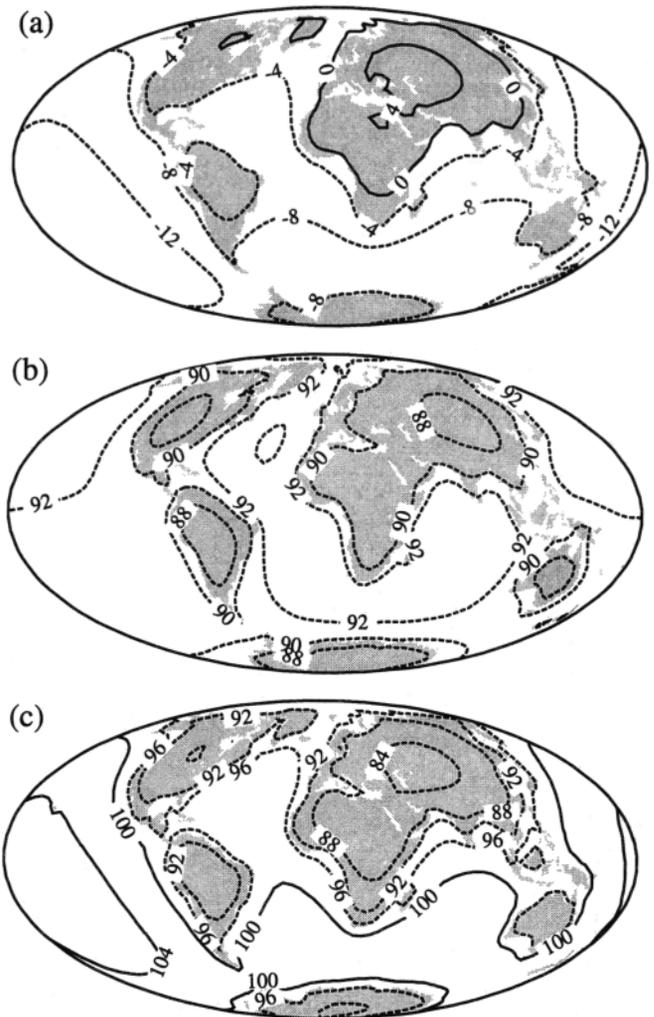


Figure 1. Calculations for the addition of water to the oceans, without regard to the continental sources of this water, given as a percentage of S_V . Shown are (a) vertical displacement of the crust, measured in an inertial reference frame, (b) absolute and (c) relative sea level change.

than average because the solid earth moves away from the ocean surface there. In contrast, the North, Mediterranean and Red seas deepen up to 15% more slowly than the global average because the solid earth moves toward them.

Mass Depletion on the Continents

Continental glacial masses are probably the major source of present-day sea level rise. Because glaciated regions are less uniformly distributed than are the oceans, we expect greater vertical uplift and significantly depressed potential height in areas of widespread deglaciation. We also expect a large translation of the solid earth toward this melted region. This combination should produce decreased absolute and relative sea level change near the deglaciated area.

We investigated the effects of continental mass depletion from three possible sources: the Antarctic and Greenland ice sheets, and mountain glaciers worldwide. In these calculations, the spatial pattern of mass depletion is proportional to the average mass balance. Although this assumption is unlikely to be the case, it should lead to the general patterns and magnitudes of crustal motion and sea level rise.

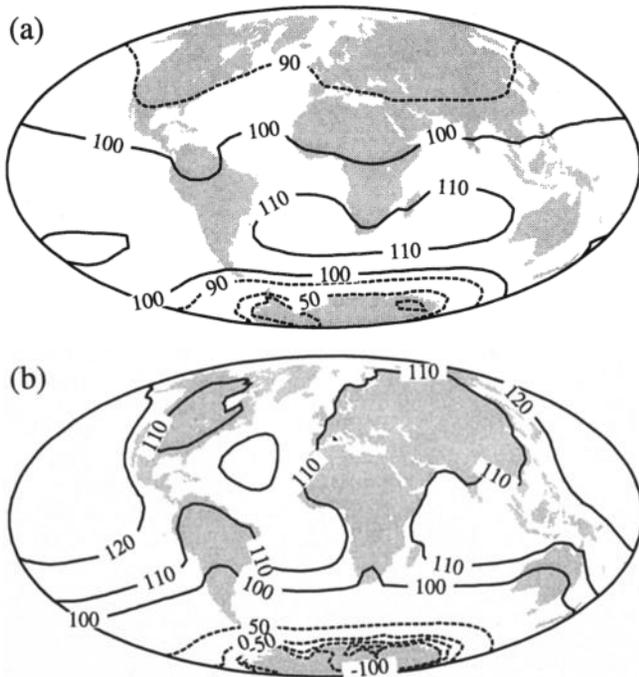


Figure 2. Calculations in which mass depletion in Antarctica is included as the sole continental source of sea level rise. Shown are (a) absolute and (b) relative sea level change. The minimum contour around Antarctica in (a) is $S_A = 0$. Vertical displacement is the difference between (a) and (b).

Antarctica

We used accumulation values of *Giovinetto and Bentley* [1985] to calculate the mass imbalance of Antarctica. For the unloading of the Antarctic ice sheet and the subsequent loading of the ocean basins, the solid earth moves rigidly toward East Antarctica ($81.6^\circ S, 42.3^\circ E$) by about 38% of S_V . This translation results in negative vertical displacements in the northern hemisphere. Close to Antarctica, vertical displacements are positive and on the order of S_V .

The southward translation of the earth causes absolute sea level rise measured in the north to be smaller than the global average (Fig. 2a). The southern hemisphere exhibits correspondingly higher values of S_A , except close to the Antarctic coast, where the potential is low due to mass loss on the adjacent continent. Because the ocean area of the southern hemisphere is larger than that of the northern, the increases in S_A in the south cause $(S_A)_O$ to increase to 99.4% of S_V (compare to Fig. 1b).

Relative sea level, which combines decreases in sea surface height and the uplift of the ocean floor near the ice sheet, occurs at rates up to S_V around Antarctica, but is of opposite sign (Fig. 2b). Decreases in the southern hemisphere are balanced over much of the rest of the globe, where relative sea level change is 10% to 20% greater than S_V .

Greenland

For the Greenland ice sheet, we used accumulation values given by *Giovinetto, pers. comm.* [1996], obtained using procedures of *Giovinetto and Zwally* [1995] and data from *Ohmura and Reeh* [1991]. The hemisphere dependence of S_A and S_R for Greenland is opposite that for Antarctica, and their values near the ice sheet are more extreme. This

is because the area over which mass depletion occurs in Greenland is smaller, causing the changes in U and Φ to be more severe. The earth moves toward eastern Greenland ($70.4^\circ N, 26.0^\circ E$) by 49% of S_V . Thus, displacements in the south are uniformly negative and only in the northern hemisphere is $S_A > S_V$ (Fig. 3a). Because the ocean area in the north is small, $(S_A)_O = 91.6\%$ of S_V . Relative sea level rise in the north is generally smaller than the global average, balanced by larger values in the south (Fig. 3b). Parts of North America and Europe experience slow or negative rates of relative sea level rise and crustal uplift rates of order S_V .

Small Mountain Glaciers

The small mountain glaciers of the world are largely located in the northern hemisphere, so the results are similar to those for Greenland. We used a compilation of the mass balances of 31 small glacier systems given by *Meier* [1984] as point sources. We spread the accumulation values for these points over nearby mountainous areas given by the *U.S. Navy Global Elevation Data* [1984]. The rigid translation of the solid earth toward northern Greenland ($85.3^\circ N, 58.8^\circ E$) is 30.5% of S_V . Again, vertical displacements are positive in the north, negative in the south and large near areas of significant mass depletion. As before, the southern hemisphere exhibits below average absolute sea level rise, causing $(S_A)_O$ to be 93.2% of S_V (Fig. 4a). Relative sea level rise (Fig. 4b) is higher than average in the south, except near the deglaciated southern Andes, and below average in the north, where it is extremely so at high latitudes.

Discussion and Conclusions

The volume of the ocean basins increases in all scenarios, by up to 8% of the total volume of sea water added. This volume change leads to a discrepancy between global averages of absolute sea level rise measured by satellites and

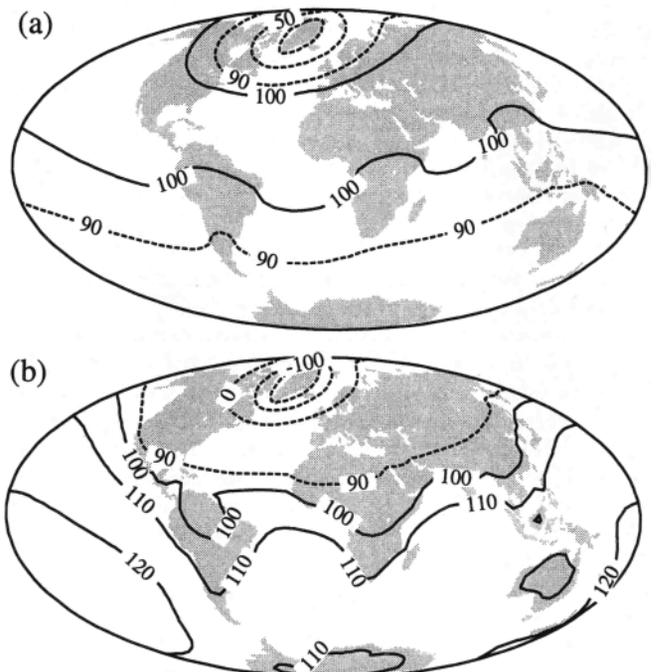


Figure 3. Similar to Fig. 2, using Greenland ice as the continental source of sea level rise. The minimum contours in Greenland are (a) $S_A = -100$ and (b) $S_R = -300$.

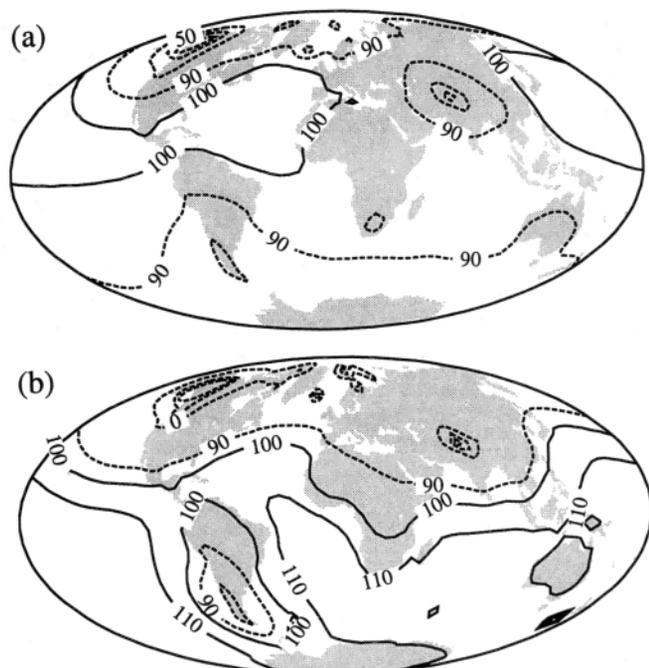


Figure 4. Similar to Fig. 2, using small mountain glaciers as the continental source of sea level rise. The minimum contours in Alaska are (a) $S_A = -100$ and (b) $S_R = -200$.

relative sea level rise measured by tide gauges, which correctly measure S_V when globally averaged. Given current measurement accuracy, a discrepancy of a few percent is not significant, although it could become so as the accuracy and duration of data sets increase. In addition, Figures 2-4 show a spatial variability in absolute sea surface height which depends significantly on the sources of sea level rise. In general, sea surface height is about 10% above average in the hemisphere of mass depletion and equally less than average in the opposite hemisphere. Care must be taken so that an unrepresentative portion of the oceans is not overly weighted in an analysis of sea surface height measurements.

Perhaps the most important consequence of this study involves the use of tide gauge data to estimate rates of global sea level rise. We found that relative sea level rise observed on continental coastlines is typically about 5% lower than the global average (Fig. 1c). Since most tide gauges are located on such coastlines, this factor could introduce a bias in the tide gauge data. Continental sources of mass depletion introduce further possible bias. Most of the tide gauge stations with records of sufficient duration are located in the northern hemisphere [Douglas, 1991]. In each of the climate change scenarios presented, there is a significant difference between the relative sea level rise measured in the northern and southern hemispheres. If Antarctica alone is losing mass, northern hemisphere stations measure relative sea rise occurring 10 to 20% faster than the global average (Fig. 2b). These rates are correspondingly slow for mass depletion in the north. Some stations near sources of continental mass depletion may seriously underestimate the global average.

We have presented results for a few extreme models of present-day climate change in which all the sea level rise originates from a single continental source. In fact, a com-

ponent of sea level rise probably originates from each of these sources and some combination of them would provide a more realistic climate change scenario. In addition, other sources of continental mass depletion may be important [e.g., Gornitz *et al.*, 1994], and the pattern of deglaciation is likely to be different than assumed here. If we knew the contribution to sea level rise from each source, it would be possible to provide a correction factor for each tide gauge to relate its measurement directly to the global average. The magnitudes of these contributions, however, are still uncertain [e.g., Warrick and Oerlemans, 1990], and it is currently impossible to do this for any tide gauge. It is possible, however, that these correction terms could be estimated from discrepancies among the tide gauge records from different regions of the world. If this can be achieved, tide gauges could provide information about the sources of continental mass depletion which have caused the observed sea level rise of the past century. Additional spatial information could be gained from satellite measurements of sea surface height and vertical displacement, although these data are of limited duration.

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References

- Douglas, B., Global Sea Level Rise, *J. Geophys. Res.*, **96**, 6981-92, 1991.
- Farrell, W. E., Deformation of the earth by surface loads, *Rev. Geophys. and Space Phys.*, **10**(3), 761-797, 1972.
- Farrell, W. E., and J. A. Clark, On postglacial sea level, *Geophys. J. R. Astron. Soc.*, **46**, 647-667, 1976.
- Giovinetto, M. and C. Bentley, Surface balance in ice drainage systems of Antarctica, *Ant. J. of the U.S.*, **20**(4), 6-13, 1985.
- Giovinetto, M. and H. Zwally, An assessment of the mass budgets of Antarctica and Greenland using accumulation derived from remotely sensed data in areas of dry snow, *Zeitschrift für Gletscherkunde und Glazialgeologie*, **31**, 25-37, 1995.
- Gornitz, V., C. Rosenzweig, and D. Hillel, Is sea level rising or falling?, *Nature*, **371**, 481, 1994.
- Hager, B. H., Weighing the ice sheets using space geodesy: A way to measure changes in ice sheet mass, *Eos Trans. AGU*, **72**(17), 91, 1991.
- James, T., and E. Ivins, Present-day Antarctic ice mass changes and crustal motion, *Geophys. Res. Lett.*, **22**, 973-6, 1995.
- Meier, M. F., Contribution of small glaciers to global sea level, *Science*, **226**, 1418-1421, 1984.
- Nerem, R. S., Measuring global mean sea level variations using TOPEX/POSEIDON altimeter data, *J. Geophys. Res.*, **100**, 25135-51, 1995.
- Ohmura, A., and N. Reeh, New precipitation and accumulation maps for Greenland, *J. of Glaciology*, **37**, 140-148, 1991.
- U.S. Navy Global Elevation Data, 10-min, obtained from the National Center for Atmospheric Research Data Support Section, Scientific Computing Division, 1984.
- Wahr, J., A. Trupin, and H. DaZhong, Predictions of vertical uplift caused by changing polar ice volumes on a viscoelastic earth, *Geophys. Res. Lett.*, **22**, 977-980, 1995.
- Warrick, R., and J. Oerlemans, Sea Level Rise, in *Climate Change: The IPCC Scientific Assessment*, ed. by J. Houghton, G. Jenkins, and J. Ephraums, pp. 260-281, Cambridge University Press, Cambridge, 1990.

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